COMPUTATIONAL HAEMODYNAMICS.
19th February 2015. Time:2:30 hrs. Prof. Dr Eleuterio Toro

Question 1. Consider the *linear* version of Kolgan's scheme (obtained as for MUSCL-Hancock but neglecting the boundary extrapolated data evolution step) for the linear advection equation

$$\partial_t q + \lambda \partial_x q = 0 \tag{1}$$

solved by the conservative method

$$q_i^{n+1} = q_i^n - \frac{\Delta t}{\Delta x}[f_{i+\frac{1}{2}} - f_{i-\frac{1}{2}}] \, . \tag{2}$$

For $\lambda > 0$ the numerical flux is

$$f_{i+\frac{1}{2}} = \lambda(q_i^n + \frac{1}{2}\Delta x \Delta_i) \, , \quad \Delta_i = \Delta_{i+\frac{1}{2}} = \frac{q_{i+1}^n - q_i^n}{\Delta x} \, . \tag{3}$$

1. Show that the resulting three-point scheme is

$$q_i^{n+1} = b_{-1} q_{i-1}^n + b_0 q_i^n + b_1 q_{i+1}^n \, , \tag{4}$$

with coefficients

$$b_{-1} = \frac{1}{2}c \, , \quad b_0 = 1 \, , \quad b_1 = -\frac{1}{2}c \, . \tag{5}$$

2. Derive the local truncation error of scheme (4)-(5) and state its order of accuracy. Comment on your results.
3. Analyse the linear stability of scheme (4)-(5) using the von Neumann method. Comment on your results.

Question 2. Show that the following schemes for the linear advection equation (1) are TVD:

1. The FORCE scheme (Recall: the FORCE flux is the arithmetic mean of LW and LF fluxes)
2. The Godunov upwind scheme for $\lambda > 0$.

Question 3. Consider the 1D blood flow equations in (physically) conservation-law form

$$\partial_t A + \partial_x(Au) = 0 \, , \quad \partial_t(Au) + \partial_x(Au^2 + \gamma A^{3/2}) = 0 \, . \tag{6}$$

1. Starting from equations (6) and assuming smooth solutions show that the equations in terms of $\mathbf{W} = [A, u]^T$ can be written as a system in conservation-law form

$$\partial_t \mathbf{W} + \partial_x \mathbf{G}(\mathbf{W}) = \mathbf{0} \tag{7}$$

specifying the flux vector $\mathbf{G}(\mathbf{W})$.

2. Comment on the fact that we now have two different systems in conservation-law form for the shallow water equations.